King Post Truss Joinery Analysis
Case Study

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Truss Loading

Basis of Design:
- Douglas Fir #1, S4S, unseasoned
- Loads are dead + snow, self-weight included
- 24' span
- 8:12 pitch
- Rafters continuously laterally braced
- Pinned connections
- No friction
Strut / Rafter Joint

- 13100#-in
- 8260#
- 980#
- 1140#
- 13100#-in

- 5x6
- 8x10
- 1140#
- 9140#
- 13100#-in
- 2300#
Strut / Rafter Joint

Failure Modes:
- Rafter Bearing on Strut
- Bending and Compression in Rafter at Reduced Section
Strut / Rafter Joint -
Rafter Bearing on Strut (Check 1)

\[ f_c = \frac{C}{A} = 126 \text{ psi} \]

\[ C = 2,120\# \]

\[ A = (1.5'' \times 5.6'') \times 2 = 16.8 \text{ in}^2 \]

\[ F'_{c'} = \min \left\{ F'_{22.6^\circ \text{ strut}} = 1,020 \text{ psi (Hankinson)} \right. \]

\[ F'_{c' \perp \text{ rafter}} = 625 \text{ psi} \]

\[ = 625 \text{ psi} > 126 \text{ psi} \therefore \text{OK} \]
Strut / Rafter Joint -
Rafter Bearing on Strut (Check 2)

Conservatively ignoring tenon bearing – some might argue for using half of it or more

\[
f_c = \frac{C}{A} = 260 \text{ psi}
\]

\[
C = 880\#
\]

\[
A = 0.75'' \times 4.5'' = 3.38 \text{ in}^2
\]

\[
F'_c = \min \left\{ \begin{array}{l}
F'_{67.4^\circ \text{ strut}} = 670 \text{ psi (Hankinson)} \\
F'_{\text{r after}} = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi}
\end{array} \right.
\]

\[
= 670 \text{ psi} > 260 \text{ psi} \therefore \text{OK}
\]
Strut / Rafter Joint - Bending and Compression in Rafter at Reduced Section

Rafter Net Section Properties:
- \( A = 58.1 \text{ in}^2 \)
- \( S = 71.5 \text{ in}^3 \)

Compression Only:
- \( f_c = \frac{C}{A} = \frac{8,260 \#}{58.1 \text{ in}^2} = 142 \text{ psi} \)
- \( F_c' = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi} \)
- \( 1,150 \text{ psi} > 142 \text{ psi} \therefore \text{OK} \)

Bending and Axial Compression Interaction:
- \( \left( \frac{f_c}{F_c'} \right)^2 + \left( \frac{f_b}{F_b'} \right)^2 = 0.15 < 1.0 \therefore \text{OK} \)
- \( f_b = \frac{M}{S} = \frac{13,100 \# \text{-in}}{71.5 \text{ in}^3} = 183 \text{ psi} \)
- \( F_b' = 1,200 \text{ psi} \times 1.15 = 1,380 \text{ psi} \)
Failure Modes:

- Strut Bearing on King Post
- Shear Through King Post Relish
- King Post Tension at Reduced Section (not checked - top of king post joint will control due to lower cross-sectional area)
- Tie Tension at Reduced Section (not checked - heel joint will control)
- King Post/Tie Peg Shear (not checked - OK by inspection)
**Strut / King Post Joint - Strut Bearing on King Post (Check 1)**

\[ f_c = \frac{C}{A} = 105 \text{ psi} \]

- \( C = 1,930 \# \)
- \( A = (1.5'' \times 6.1'') \times 2 = 18.3 \text{ in}^2 \)

\[ F'_c = \min \left\{ F'_{33.7°, \text{strut}} = 914 \text{ psi (Hankinson)} \right\} \]

\[ F'_{c_{\perp \text{king post}}} = 625 \text{ psi} \]

\[ = 625 \text{ psi} > 105 \text{ psi} : \text{OK} \]
Strut / King Post Joint - Strut Bearing on King Post (Check 2)

\[ f_c = \frac{C}{A} = 382 \, \text{psi} \]

\[ C = 1,290\# \]

\[ A = 0.75'' \times 4.5'' = 3.38 \, \text{in}^2 \]

\[ F'_c = \min \left\{ \begin{array}{c} F'_{56.3^\circ \text{strut}} = 727 \, \text{psi} \, \text{(Hankinson)} \\ F'_{c \text{king post}} = 1,000 \, \text{psi} \times 1.15 = 1150 \, \text{psi} \end{array} \right\} = 727 \, \text{psi} > 382 \, \text{psi} \therefore \text{OK} \]
Strut / King Post Joint - Shear Through King Post Relish

\[ f_v = \frac{V}{A} = 36 \text{ psi} \]

\[ V = 1,290\# \]

\[ A = (0.75" \times 2 + 4.5") \times 6" = 36 \text{ in}^2 \]

\[ F'_v = 170 \text{ psi} \times 1.15 \]

\[ = 196 \text{ psi} > 36 \text{ psi} \quad \therefore \text{OK} \]
Failure Modes:
- Ridge Beam Bearing on King Post
- King Post Bearing on Rafters
- Shear Through King Post Relish
- King Post Tension at Reduced Section
- Rafter Compression over Effective Area at Connection
Rafter / King Post Joint - Ridge Beam Bearing on King Post

\[ f_c = \frac{C}{A} = 500 \text{ psi} \]

\[ C = 2,000\# \]

\[ A = (1.25'' - 0.25'' \text{ shrinkage}) * (5.5'' - 1.5'') = 4.00 \text{ in}^2 \]

\[ F_c' = \min \left\{ \begin{array}{l} F_{c\perp \text{ridge}}' = 625 \text{ psi} \\ F_{c\text{king post}}' = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi} \end{array} \right\} = 625 \text{ psi} > 500 \text{ psi} \therefore \text{OK} \]
Rafter / King Post Joint -
King Post Bearing on Rafters (Check 1)

\[ f_c = \frac{C}{A} = 170 \text{ psi} \]

\[ C = 5,970\# + 350\# = 6,320\# \]

\[ A = (6.75" \times 2.75") \times 2 = 37.1 \text{ in}^2 \]

\[ F_c' = \min \left\{ F'_{c_{\perp \text{king post}}} = 625 \text{ psi} \right\} \]

\[ F'_{33.7^\circ \text{rafter}} = 914 \text{ psi (Hankinson)} \]

\[ = 625 \text{ psi} > 170 \text{ psi} \quad \therefore \text{OK} \]
\[ f_c = \frac{C}{A} = 461 \text{ psi} \]

\[ C = 2,920\# + 2,000\# + 2,000\# = 6,920\# \]

\[ A = (1" \times 7.5")^2 = 15 \text{ in}^2 \]

\[ F'_c = \min \left\{ \begin{array}{l} F'_{\text{king post}} = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi} \\ F'_{56.3^\circ \text{rafter}} = 727 \text{ psi} \ (\text{Hankinson}) \end{array} \right\} = 727 \text{ psi} > 461 \text{ psi} \therefore \text{OK} \]
Conservatively neglecting rafter tenons

\[ f_v = \frac{V}{A} = 100 \text{ psi} \]

\[ V = 2,920\# + 2,000\# + 2,000\# = 6,920\# \]

\[ A = (4.6'' \times 7.5'') \times 2 = 69.0 \text{ in}^2 \]

\[ F'_v = 170 \text{ psi} \times 1.15 \]

\[ = 196 \text{ psi} > 100 \text{ psi} \therefore \text{OK} \]
Rafter / King Post Joint -
King Post Tension at Reduced Section

King Post Cross Section

\[ f_t = \frac{T}{A} = 364 \text{ psi} \]

\[ T = 2,920\# + 2,000\# + 2,000\# = 6,920\# \]

\[ A = 19.0 \text{ in}^2 \]

\[ F_t' = 825 \text{ psi} \times 1.15 \]

\[ = 949 \text{ psi} > 364 \text{ psi} \therefore \text{OK} \]
Rafter / King Post Joint -
Rafter Compression Over Effective
Area at Connection

\[ f_c = \frac{C}{A} = 157 \text{ psi} \]

\[ C = 7,180 \# \]

\[ A = 6.1" \times 7.5" = 45.8 \text{ in}^2 \]

\[ F'_c = 1,000 \text{ psi} \times 1.15 \]

\[ = 1,150 \text{ psi} > 157 \text{ psi} \therefore \text{OK} \]
Heel Joint

1000# (plate not shown)

8x10

10480#

870#

8x12

160#

8240#

8x12

8x8

8690#

8x10

6x12

1000#
Heel Joint

1000# (plate not shown)

8x10

8x12

160#

Failure Modes:

- Plate Bearing on Post
- Rafter Bearing on Tie
- Shear Through Tie Relish
- Tie Tension at Reduced Section
- Tie Bearing on Post
Heel Joint - Plate Bearing on Post

$1,000 \# \text{ (plate not shown)}$

\[ f_c = \frac{C}{A} = 242 \text{ psi} \]

\[ C = 1,000\# \]

\[ A = (1" - 0.25" \text{ shrinkage}) \times (5.5") \]

\[ = 4.13 \text{ in}^2 \]

\[ F_c' \]

\[ = \min \left\{ \frac{F_{c\perp \text{ plate}}'}{F_{c\perp \text{ post}}} \right\} \]

\[ = 625 \text{ psi} \]

\[ = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi} \]

\[ = 625 \text{ psi} > 242 \text{ psi} \therefore \text{OK} \]
Heel Joint - 
Rafter Bearing on Tie

\( f_c = \frac{C}{A} = 793 \text{ psi} \)

\( C = 10,480\# \)

\( A = (2.4" \times 2.75") \times 2 = 13.2 \text{ in}^2 \)

\( F_c' = \min \left\{ \begin{array}{l}
F_{33.7\,\text{tie}}' = 875 \text{ psi (hankinson)} \\
F_{\text{crafter}}' = 1,000 \text{ psi} \times 1.15 = 1,150 \text{ psi}
\end{array} \right. 
\)

\( 875 \text{ psi} > 793 \text{ psi} \therefore \text{OK} \)
Heel Joint - Shear Through Tie Relish

\[ f_v = \frac{V}{A} = 109 \text{ psi} \]

\[ V = 8,720\# \]

\[ A = 79.8 \text{ in}^2 \]

\[ F_v' = 170 \text{ psi} \times 1.15 \]

\[ = 196 \text{ psi} > 109 \text{ psi} : OK \]
Heel Joint -
Tie Tension at Reduced Section

\[ f_t = \frac{T}{A} = 122 \text{ psi} \]

\[ T = 8,720\# \]

\[ A = 9.5'' \times 7.5'' = 71.3 \text{ in}^2 \]

\[ F_t' = 675 \text{ psi} \times 1.15 \]

\[ = 776 \text{ psi} > 122 \text{ psi} \therefore \text{OK} \]
Heel Joint - Tie Bearing on Post

1000# (plate not shown)

\[ f_c = \frac{C}{A} = 221 \text{ psi} \]

\[ C = 5810\# + 720\# + 160\# = 6690\# \]

\[ A = 30.3 \text{ in}^2 \]

\[ F'_c = \min\left\{ \begin{array}{l} F'_{c, \text{tie}} = 625 \text{ psi} \\ F'_{c, \text{post}} = 1000 \text{ psi} \times 1.15 = 1150 \text{ psi} \end{array} \right\} = 625 \text{ psi} > 221 \text{ psi} \therefore \text{OK} \]
Thank You

Questions?
Comments?